

What Is Claimed Is:

1 1. A method for using a computer system to solve a system of
2 nonlinear equations specified by a vector function, \mathbf{f} , wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents a
3 set of nonlinear equations, $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, wherein \mathbf{x}
4 is a vector $(x_1, x_2, x_3, \dots, x_n)$, the method comprising:
5 receiving a representation of a subbox $\mathbf{X} = (X_1, X_2, \dots, X_n)$, wherein for
6 each dimension, i , the representation of X_i includes a first floating-point number,
7 a_i , representing the left endpoint of X_i , and a second floating-point number, b_i ,
8 representing the right endpoint of X_i ;
9 storing the representation in a computer memory;
10 applying term consistency to the set of nonlinear equations, $f_1(\mathbf{x}) = 0$,
11 $f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, over \mathbf{X} , and excluding portions of \mathbf{X} that violate
12 any of these nonlinear equations;
13 applying box consistency to the set of nonlinear equations over \mathbf{X} , and
14 excluding portions of \mathbf{X} that violate any of the nonlinear equations; and
15 performing an interval Newton step on \mathbf{X} to produce a resulting subbox \mathbf{Y} ,
16 wherein the point of expansion of the interval Newton step is a point \mathbf{x} within \mathbf{X} ,
17 and wherein performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using
18 interval arithmetic to produce an interval result $\mathbf{f}^I(\mathbf{x})$.

1 2. The method of claim 1, wherein performing the interval Newton
2 step involves:
3 computing $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
4 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
5 determining if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

1 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
2 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 3. The method of claim 2, further comprising:
2 applying term consistency to the preconditioned set of nonlinear equations
3 $\mathbf{B}\mathbf{f}(\mathbf{x}) = \mathbf{0}$ over the subbox \mathbf{X} ; and
4 excluding portions of \mathbf{X} that violate the preconditioned set of nonlinear
5 equations.

1 4. The method of claim 2, further comprising:
2 applying box consistency to the preconditioned set of nonlinear equations
3 $\mathbf{B}\mathbf{f}(\mathbf{x}) = \mathbf{0}$ over the subbox \mathbf{X} ; and
4 excluding portions of \mathbf{X} that violate the preconditioned set of nonlinear
5 equations.

1 5. The method of claim 1, wherein applying term consistency to the
2 set of nonlinear equations involves:
3 for each nonlinear equation $f_i(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$,
4 symbolically manipulating $f_i(\mathbf{x}) = 0$ to solve for an invertible term, $g(x'_j)$, thereby
5 producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein $g(x'_j)$ can be analytically
6 inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
7 substituting the subbox \mathbf{X} into the modified equation to produce the
8 equation $g(X'_j) = h(\mathbf{X})$;
9 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
10 intersecting X'_j with the vector element X_j to produce a new subbox \mathbf{X}^+ ;

11 wherein the new subbox \mathbf{X}^+ contains all solutions of the system of
12 equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the subbox \mathbf{X} , and wherein the width of the new subbox
13 \mathbf{X}^+ is less than or equal to the width of the subbox \mathbf{X} .

1 6. The method of claim 1, further comprising:
2 evaluating a first termination condition, wherein the first termination
3 condition is TRUE if,
4 zero is contained within $\mathbf{f}^1(\mathbf{x})$,
5 $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the
6 function \mathbf{f} evaluated as a function of \mathbf{x} over the subbox \mathbf{X} , and
7 the solution \mathbf{Y} of $\mathbf{M}(\mathbf{x}, \mathbf{X}) (\mathbf{y} - \mathbf{x}) = \mathbf{r}$ contains \mathbf{X} ; and
8 if the first termination condition is TRUE, terminating and recording \mathbf{X} as
9 a final bound.

1 7. The method of claim 6, wherein the method further comprises:
2 evaluating a second termination condition;
3 wherein the second termination condition is TRUE if a function of the
4 width of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the width of the
5 function \mathbf{f} over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and
6 if the second termination condition is TRUE, terminating and recording \mathbf{X}
7 as a final bound.

1 8. A computer-readable storage medium storing instructions that
2 when executed by a computer cause the computer to perform a method for using a
3 computer system to solve a system of nonlinear equations specified by a vector
4 function, \mathbf{f} , wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents a set of nonlinear equations, $f_i(\mathbf{x}) = 0$,

5 $f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, wherein \mathbf{x} is a vector $(x_1, x_2, x_3, \dots, x_n)$, the
 6 method comprising:
 7 receiving a representation of a subbox $\mathbf{X} = (X_1, X_2, \dots, X_n)$, wherein for
 8 each dimension, i , the representation of X_i includes a first floating-point number,
 9 a_i , representing the left endpoint of X_i , and a second floating-point number, b_i ,
 10 representing the right endpoint of X_i ;
 11 storing the representation in a computer memory;
 12 applying term consistency to the set of nonlinear equations, $f_1(\mathbf{x}) = 0$,
 13 $f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, over \mathbf{X} , and excluding portions of \mathbf{X} that violate
 14 any of these nonlinear equations;
 15 applying box consistency to the set of nonlinear equations over \mathbf{X} , and
 16 excluding portions of \mathbf{X} that violate any of the nonlinear equations; and
 17 performing an interval Newton step on \mathbf{X} to produce a resulting subbox \mathbf{Y} ,
 18 wherein the point of expansion of the interval Newton step is a point \mathbf{x} within \mathbf{X} ,
 19 and wherein performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using
 20 interval arithmetic to produce an interval result $\mathbf{f}^I(\mathbf{x})$.

1 9. The computer-readable storage medium of claim 8, wherein
 2 performing the interval Newton step involves:
 3 computing $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
 4 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
 5 determining if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
 6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
 7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
 8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 10. The computer-readable storage medium of claim 9, wherein the
 2 method further comprises:
 3 applying term consistency to the preconditioned set of nonlinear equations
 4 $\mathbf{Bf}(\mathbf{x}) = \mathbf{0}$ over the subbox \mathbf{X} ; and
 5 excluding portions of \mathbf{X} that violate the preconditioned set of nonlinear
 6 equations.

1 11. The computer-readable storage medium of claim 9, wherein the
 2 method further comprises:
 3 applying box consistency to the preconditioned set of nonlinear equations
 4 $\mathbf{Bf}(\mathbf{x}) = \mathbf{0}$ over the subbox \mathbf{X} ; and
 5 excluding portions of \mathbf{X} that violate the preconditioned set of nonlinear
 6 equations.

1 12. The computer-readable storage medium of claim 8, wherein
 2 applying term consistency to the set of nonlinear equations involves:
 3 for each nonlinear equation $f_i(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$,
 4 symbolically manipulating $f_i(\mathbf{x}) = 0$ to solve for an invertible term, $g(x'_j)$, thereby
 5 producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein $g(x'_j)$ can be analytically
 6 inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
 7 substituting the subbox \mathbf{X} into the modified equation to produce the
 8 equation $g(X'_j) = h(\mathbf{X})$;
 9 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
 10 intersecting X'_j with the vector element X_j to produce a new subbox \mathbf{X}^+ ;
 11 wherein the new subbox \mathbf{X}^+ contains all solutions of the system of
 12 equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the subbox \mathbf{X} , and wherein the width of the new subbox
 13 \mathbf{X}^+ is less than or equal to the width of the subbox \mathbf{X} .

1 13. The computer-readable storage medium of claim 8, wherein the
2 method further comprises:
3 evaluating a first termination condition, wherein the first termination
4 condition is TRUE if,
5 zero is contained within $\mathbf{f}^l(\mathbf{x})$,
6 $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the
7 function \mathbf{f} evaluated as a function of \mathbf{x} over the subbox \mathbf{X} , and
8 the solution \mathbf{Y} of $\mathbf{M}(\mathbf{x}, \mathbf{X}) (\mathbf{y} - \mathbf{x}) = \mathbf{r}$ contains \mathbf{X} ; and
9 if the first termination condition is TRUE, terminating and recording \mathbf{X} as
10 a final bound.

1 14. The computer-readable storage medium of claim 13, wherein the
2 method further comprises:
3 evaluating a second termination condition;
4 wherein the second termination condition is TRUE if a function of the
5 width of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the width of the
6 function \mathbf{f} over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and
7 if the second termination condition is TRUE, terminating and recording \mathbf{X}
8 as a final bound.

1 15. An apparatus that solves a system of nonlinear equations specified
2 by a vector function, \mathbf{f} , wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents a set of nonlinear equations,
3 $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, wherein \mathbf{x} is a vector $(x_1, x_2, x_3, \dots, x_n)$,
4 the apparatus comprising:
5 a receiving mechanism that is configured to receive a representation of a
6 subbox $\mathbf{X} = (X_1, X_2, \dots, X_n)$, wherein for each dimension, i , the representation of

7 X_i includes a first floating-point number, a_i , representing the left endpoint of X_i ,
 8 and a second floating-point number, b_i , representing the right endpoint of X_i ;
 9 a computer memory for storing the representation;
 10 a term consistency mechanism that is configured to apply term consistency
 11 to the set of nonlinear equations, $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$, over
 12 \mathbf{X} , and to exclude portions of \mathbf{X} that violate any of these nonlinear equations;
 13 a box consistency mechanism that is configured to apply box consistency
 14 to the set of nonlinear equations over \mathbf{X} , and to exclude portions of \mathbf{X} that violate
 15 any of the nonlinear equations; and
 16 an interval Newton mechanism that is configured to perform an interval
 17 Newton step on \mathbf{X} to produce a resulting subbox \mathbf{Y} , wherein the point of
 18 expansion of the interval Newton step is a point \mathbf{x} within \mathbf{X} , and wherein
 19 performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using interval
 20 arithmetic to produce an interval result $\mathbf{f}^I(\mathbf{x})$.

1 16. The apparatus of claim 15, wherein the interval Newton
 2 mechanism is configured to:
 3 compute $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f} evaluated
 4 as a function of \mathbf{x} over the subbox \mathbf{X} ; and
 5 determine if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
 6 that contain the values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
 7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
 8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 17. The apparatus of claim 16, wherein the term consistency
 2 mechanism is configured to:

1 apply term consistency to the preconditioned set of nonlinear equations
2 **Bf(x) = 0** over the subbox **X**; and to
3 exclude portions of **X** that violate the preconditioned set of nonlinear
4 equations.

1 18. The apparatus of claim 16, wherein the box consistency
2 mechanism is configured to:
3 apply box consistency to the preconditioned set of nonlinear equations
4 **Bf(x) = 0** over the subbox **X**; and to
5 exclude portions of **X** that violate the preconditioned set of nonlinear
6 equations.

1 19. The apparatus of claim 15, wherein for each nonlinear equation
2 $f_i(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, the term consistency mechanism is
3 configured to:
4 symbolically manipulate $f_i(\mathbf{x})=0$ to solve for an invertible term, $g(x'_j)$,
5 thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein $g(x'_j)$ can be
6 analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;
7 substitute the subbox **X** into the modified equation to produce the equation
8 $g(X'_j) = h(\mathbf{X})$;
9 solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to
10 intersect X'_j with the vector element X_j to produce a new subbox \mathbf{X}^+ ;
11 wherein the new subbox \mathbf{X}^+ contains all solutions of the system of
12 equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the subbox **X**, and wherein the width of the new subbox
13 \mathbf{X}^+ is less than or equal to the width of the subbox **X**.

1 20. The apparatus of claim 15, further comprising a termination
2 mechanism that is configured to:
3 evaluate a first termination condition, wherein the first termination
4 condition is TRUE if,
5 zero is contained within $\mathbf{f}^l(\mathbf{x})$,
6 $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the
7 function \mathbf{f} evaluated as a function of \mathbf{x} over the subbox \mathbf{X} , and
8 the solution \mathbf{Y} of $\mathbf{M}(\mathbf{x}, \mathbf{X}) (\mathbf{y} - \mathbf{x}) = \mathbf{r}$ contains \mathbf{X} ; and to
9 terminate and record \mathbf{X} as a final bound if the first termination condition is
10 TRUE.

1 21. The apparatus of claim 20, wherein the termination mechanism is
2 additionally configured to:
3 evaluate a second termination condition;
4 wherein the second termination condition is TRUE if a function of the
5 width of the subbox \mathbf{X} is less than a pre-specified value, ε_X , and the width of the
6 function \mathbf{f} over the subbox \mathbf{X} is less than a pre-specified value, ε_F ; and to
7 terminate and record \mathbf{X} as a final bound if the second termination
8 condition is TRUE.